

Q No

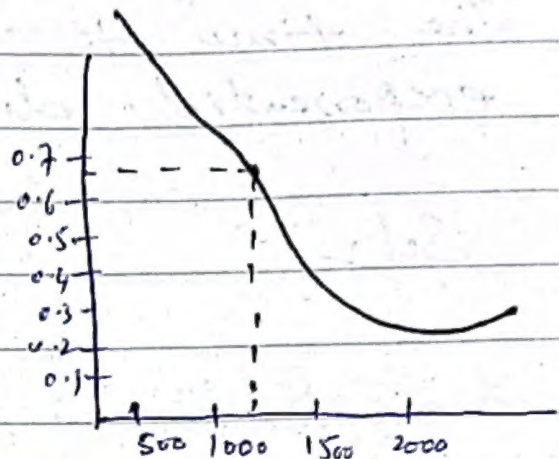
we have tested 20x unit and found that our MTBF is 2996 Hrs what is the reliability of product @ 1200 hr of operation.

Data::

$$t = 1200$$

$$\theta = 2996$$

We know that



$$MTTF = \theta = \frac{\text{operation Time}}{\text{No. of failure}}$$

$$MTBF = \lambda = \frac{\text{No of failure}}{\text{operation time}}$$

$$\theta = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{\theta}$$

$$\begin{aligned} \text{Reliability} &= R(t) = e^{-\lambda t} \\ R(t) &= e^{-\left(\frac{t}{\theta}\right)} \\ &= e^{-\left(\frac{1200}{2996}\right)} \end{aligned}$$

$$R(t) = 0.67 \approx 67\%$$

$$\text{Failure rate} = F = 1 - R$$

$$= 1 - 67\%$$

$$F = 33\% \text{ (unreliable rate)}$$



Q No 2

what is the highest failure rate for a product if it is to have a probability of survival (i.e. successful operation) of 95% at 4000 hr? Assume the time to failure follows an exponential distribution.

Sol:-

$$R(t) = e^{-\lambda t}$$

$$0.95 = e^{-[4000 \lambda]}$$

taking  $\ln$  on both side

$$\ln(0.95) = \ln e^{-[4000 \lambda]}$$

$$-0.0512 = -4000 \lambda \times \ln e$$

$$\ln e = 1$$

$$-0.0512 = -4000 \lambda$$

$$\lambda = \frac{-0.0512}{-4000}$$

$$\lambda = 12.8 \times 10^{-6}$$



48  
Imported

Example: 20 units are put on test & run at their normal operating condition for 1000 hrs. 6 of these units fail at the following hours (550, 480, 680, 790, 860, 620) what is the mean time to failure of the product:

Solution:

$$\text{Un Reliability } f = \frac{6}{20} = 0.3 \approx 30\%$$

$$\text{Reliability } (R) = 1 - 0.3 = 0.70 \approx 70\%$$

$$MTTF = \theta = \frac{(1000 \times 14) + 550 + 480 + 680 + 790 + 860 + 620}{6}$$

$$\theta = \frac{17980}{6} = 2996 \text{ Hrs to failure}$$

$$\theta = \frac{1}{\lambda} = \lambda^{-1} = \frac{1}{0.00033}$$

$$\lambda = 0.00033$$

Weibull distribution:

Q.No: 1. A machine is known to have a Weibull distribution with a shape parameter  $\beta = 2$  and a scale parameter  $\theta = 8000$ .

Find the reliability of the machine at  $t = 5000$  hours.

Given:  $\beta = 2$ ,  $\theta = 8000$ ,  $t = 5000$

Find: Reliability  $R(t)$  at  $t = 5000$  hours.

Solution: The reliability function for a Weibull distribution is given by:

$R(t) = e^{-\left(\frac{t}{\theta}\right)^\beta}$

Data:

$$t = 5000$$

$$\theta = 8000$$

$$R(t) = ??$$

$$\beta = 2$$

$$\text{Reliability} = R(t) = e^{-\left(\frac{5000}{8000}\right)^2}$$

$$= e^{-0.3906}$$

$$R(5000) = 0.676 \approx 67\%$$



Imported:

Final Exam 2024:

**Problem:-** Twenty components with CFR (constant failure rate) was observed being used in a highly stressed environment. After 25 hrs of use seven of them failed at time in hr.

2.1, 8.3, 10.9, 15.2, 16.3, 20.5, 23.8

while the remaining still functioning. Calculate the following

- (a) MTTF
- (2) Failure rate
- (3) The time which reliability 95%.
- (4) The reliability of 50 hrs of use.

$$\begin{aligned}\textcircled{1} \text{ MTTF} &= \frac{(13 \times 25) + 2.1 + 8.3 + 10.9 + 15.2 + 16.3 + 20.5 + 23.8}{7} \\ &= \frac{422.1}{7} = 60.3 \text{ hrs/failure}\end{aligned}$$

$\textcircled{2}$  Failure rate:

$$\lambda = \frac{1}{9} = \frac{1}{60.3}$$

$$= 0.0166 \text{ Failure/hr.}$$

Q3

Time at which reliability is 95%

$$R(t) = e^{-\lambda t}$$

$$0.95 = e^{-0.0166t}$$

taking  $\ln$  on both side

$$\ln(0.95) = -[0.0166\lambda t] \ln e$$

$$\ln e = 1$$

$$-0.0512 = -0.0166 \times t$$

$$t = \frac{0.0512}{0.0166}$$

$$\boxed{t = 3.09 \text{ hrs}}$$

Q4 Reliability after 50 hrs of use

$$R(t) = e^{-\lambda t}$$

$$R(50) = e^{-0.0166 \times 50}$$

$$R(50) = e^{-0.83} = 0.436$$

$$\boxed{R(50) = 43.6 \%}$$



Q.No:

A Six Sided die is tossed

- a) what is the probability of getting a "2"
- b) " " " " " number that is almost "4"
- c) " " " " " 3 or 5
- d) " " " " " No. that is greater than "3"
- e) " " " " " Getting No. that is less than or equal to 5"

Solution:-

$$S.S = \{1, 2, 3, 4, 5, 6\}$$

a) Probability getting "2"

$$P(2) = \frac{1}{6} = 0.166 \approx 16.6\%$$

b) Probability No. that is almost "4"

$$S.S = \{1, 2, 3, 4\}$$

$$P(x \leq 4) = \frac{4}{6} = \frac{2}{3} = 0.666 \approx 66.66\%$$

c) Probability 3 or 5

$$P(3 \text{ or } 5) = \frac{2}{6} = \frac{1}{3} = 0.333 \approx 33.3\%$$

① Probability of greater than 3  
 $\{4, 5, 6\}$ .

$$P(X > 3) = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$$

(e) Less than equal 5

$$S.S = \{1, 2, 3, 4, 5, 6\}$$

$$P(X \leq 5) = \frac{5}{6} = 0.83 = 83\%$$



Qryo.

A Jar contains 7 Red marbles  
6 green marbles 5 blue marble  
and 2 yellow marbles:-

- ① what is the probability of selecting green marbles.
- ②  $n$   $n$   $r$   $r$   $n$  blue marble
- ③ Green or yellow
- ④ Red then blue with replacement
- ⑤ Red then blue without replacement
- ⑥ what is the probability of selecting Red then green marble or getting a blue marble and then yellow marble with replacement.

Sol:	color	marble Qty
	Red (R) =	7
	Green (G) =	6
	Blue (B) =	5
	Yellow (Y) =	2
	Total =	<u>20</u>

④ probability of green marble

$$P(C_1) = \frac{6}{20} = \frac{3}{10} = 0.3 = 30\%$$



Not Selecting the green marble.

$$= 1 - 0.3 = 0.70 = 70\%$$

② Probability of selecting blue marble

$$P(B) = \frac{5}{20} = \frac{1}{4} = 0.25 \approx 25\%$$

③ Probability of selecting Green & Yellow

$$P(G) + P(Y)$$

$$\frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5} = 0.4 = 40\%$$

There <sup>is</sup> 40% chance of selecting  
G or yellow marbles.

④ Red then Blue with replacement it  
means once you take out red marble  
on 1st try you put it back for  
2nd try

$$= P(R) \cdot P(B)$$

$$P(R \text{ then } B) = P(R) \cdot P(B)$$

$$= \frac{7}{20} \times \frac{5}{20} = 0.0875 \approx 8.75\%$$



5) Red then blue without replacement its mean once you take out red marble on 1st try you put separat (other basket) and then try.

$$P(R) \cdot P(B)$$

$$= \frac{7}{20} \cdot \frac{5}{19} = 0.0921 \approx 9.21\%$$

6) And mean both No. will be multiply  
 $P(R) \times P(B)$

$$\frac{7}{20} \times \frac{5}{19} = 0.0921 \approx \text{60\% } 9.20\%$$

Test.  
Oct = 25 (10)

✓  
Example:

What is the highest failure rate for a product if it is to have a probability of survival (i.e. successful operation) of 75% at 4000 hrs? Assume that the time to failure follows an exponential distribution.

Solution:

$$\text{Reliability} = R(t) = e^{-\lambda t}$$

$$0.75 = e^{-\lambda(4000)}$$

$$\text{Failure rate} = \lambda = \frac{1}{\theta}$$

$$5\% = \lambda = \frac{1}{4000}$$

$$\theta = 200$$



Log	ln
Log <sub>10</sub>	Ln

$$\ln e = 1$$

ln  $\rightarrow$  Natural log.

Solution:

$$R(t) = e^{-\lambda t}$$

$$\frac{95}{100} = e^{-[\lambda \times 4000]}$$

$$0.95 = e^{-\lambda \times 4000}$$

$$\lambda = 12.8 / 10^6 \text{ hrs.}$$

$$\lambda = 0.0000128 / \text{hrs.}$$

Thus the highest failure rate is  $12.8 / 10^6$  hours for a reliability of 0.95 @ 4000 hrs.

~~Q2~~

$$e^{-[4000\lambda]}$$

$$0.95 = e^{-[4000\lambda]}$$

Taking ln on both side

$$\ln(0.95) = \ln e^{-[4000\lambda]}$$

$$\log a^b = b \log a$$

$$-0.05129 = -4000\lambda \times \ln e$$

$$0.05129 = 4000\lambda \times 1$$

$$\frac{0.05129}{4000} = \lambda$$

$$\lambda = 12.8 \times 10^{-6}$$

Final Product  
✓ 200

Example: 20 units are put on test & run at their normal operating condition for 1000 hours. 6 of those units fail at the following hours (550, 480, 6800, 780, 860, 620). What is the mean time to failure of the product.

Solution:-

MTTF and MTBF are reflection of the reliability of your product -  $\theta$  (theta)

Failure rate ( $\lambda$ ) =  $\frac{\text{No. of Failure}}{\text{operating Time}}$

MTTF & MTBF:  $\theta = \frac{\text{operating Time}}{\text{No. of Failure}}$

Un Reliability =  $\frac{6}{20} = 0.3$

the reliability =  $1 - 0.3 = 0.7$   
Reliability = 70%



Right data: Censor data left

Minitab:

$$= \frac{10000}{}$$

$$\theta = \frac{(1000 \times 14) + 550 + 400 + 680 + 790 + 860 + 620}{6}$$

$$= 898$$

~~898~~

$$\frac{17980}{6}$$

$$\theta = 2996 \text{ Hrs to failure}$$

Failure rate

$$\theta = \frac{1}{\lambda} \quad \lambda = \frac{1}{2996}$$

$$\text{Failure rate } \lambda = 0.00033$$

$$\text{MTTF} = \frac{1}{\text{Failure rate}}$$

$$= \frac{1}{0.00033} =$$

$$\text{MTTF} = 2996 \text{ Hrs.}$$

$$\text{Slope} = \frac{\text{raise}}{\text{sum}} = \frac{0}{\text{Raise}} = \text{---} y_{2\max.}$$
$$\text{Slope} = \frac{\text{raise}}{\text{sum}} = \frac{\text{Rm}}{0} = 40 \quad |$$



CFD = constant failure distribution  
exponential distribout

# Weibull Distribution

20/8/14.

You have collected on component and calculated that your product fit a weibull distribution.

$$\text{Reliability} = R(t) = e^{-\left(\frac{t}{\theta}\right)^B}$$

$$5000 = e^{-\left(\frac{5000}{8000}\right)^2} = e^{-0.3906}$$

$$R(5000) = 0.6766$$

= 67.7% Reliability

$$B = 2$$

$$\theta = 8000$$

$$T = 5000$$



✓ ~~20.5~~ CFR = constant failure rate

✓ Bath tube curve. Explaining.

✓ ~~20.5~~ Final Exam 2084 (7).

Twenty components with CFR (constant failure rate) was observed being used in a high stressed environment. After 25 hours of use seven of them failed at time in hours.

2.1, 8.3, 10.9, 15.2, 16.3, 20.5  
23.8

while the remaining is still functioning calculate the following parameters.

- ① MTTF
- ② the failure rate
- ③ the time at which the reliability is 85%.
- ④ the reliability of 50 hours of use.

① ..

$$MTTF = \frac{(13 \times 25) + 2.1 + 8.3 + 10.9 + 15.2 + 16.3 + 20.5 + 23.8}{7}$$
$$= \frac{(13 \times 25) + 97.1}{7} = \frac{422.1}{7}$$

$$= 60.3$$

② failure rate

$$\theta_2 = \frac{1}{\lambda}$$

$$= \frac{\phi}{\cancel{60.3}} \frac{1}{60.3}$$

$$= \cancel{0.0018}$$

$$\lambda = 0.0165$$

$$\lambda = 1.65$$

③

$$\text{Reliability} = e^{-\lambda t}$$

$$95\% = e^{-0.0165t}$$

taking  $\ln$  on both sides

$$\ln(0.95) = -0.0165t \ln e$$

$$-0.051 = -0.0165t$$

$$t = \frac{0.051}{0.016}$$

$$t = 3.20$$



④

Reliability = 50 Hrs.

$$R(t) = e^{-\lambda t}$$

$$= e^{0.0165(50)}$$

$$= e^{0.825}$$

$$\boxed{R(t) = 2.28} \quad R(t) = 0.438$$

Problem (2024) (8)

Imported problem solve by teacher

(i) MTT F

It is the sum of all test time divided by # of failure.

$$MTTF = \frac{2.1 + 8.3 + 10.9 + 15.2 + 18.3 + 20.5 + \frac{13 \times 25}{2}}{7}$$

$$= \frac{422.1}{7} = 60.3 \text{ hours/failure}$$

(ii) Failure rate

$$\lambda = \frac{1}{\theta} = \frac{1}{60.3}$$

$$\lambda = 0.0166 \frac{\text{Failure}}{\text{hrs.}}$$

(iii) time @ which Reliability is 95?

$$R(t) = e^{-\lambda t}$$
$$0.95 = e^{-0.0166t}$$



$$\ln 0.95 = [-0.0166t] \times 1$$

$$-0.0513 = -0.0166t$$

$$t = \frac{0.0513}{0.0166}$$

$$t = 3.09 \text{ hours}$$

(iv) Reliability after 50 hours of use

$$R(50) = e^{-0.0166 \times 50} = e^{(-0.0166 \times 50)}$$

$$R(50) = 43.6\%$$

## Probability:

which estimating that probability of a complex system like Aircraft carrier or an oil-rig it is impractical to test those system to distribution. So the reliability of these system are calculated by estimating the reliability of individual components

### Rule #1

If  $A$  &  $B$  are two events of interest and  $P_A$  &  $P_B$  are their respective probabilities of them occurring then if  $A$  &  $B$  are independent

$$P(A \& B) = P_A \times P_B$$

### Rule #2.

If  $A$  &  $B$  are mutually ~~exclusive~~ <sup>exclusive</sup> the

on only one will be available at same time

$$P(A \text{ or } B) = P_A + P_B$$



### Rule #3

20  $A_1, A_2, A_3, \dots, A_n$   
are mutually exclusive event and they  
describe all possible outcomes in a  
particular situation then.

$$P_1 + P_2 + P_3 + \dots + P_n = 1$$

### Rule #4

20 there are only two possible  
outcomes (say) Success & failure

$$P(\text{Success}) = 1 - P(\text{Failure}).$$

Note

$$P(A) = \frac{\text{\# of Favourable outcomes}}{\text{Total Possible \# of outcome}}$$

So the reliability of event A occurring.

is

$$0 \leq P(A) \leq 1$$

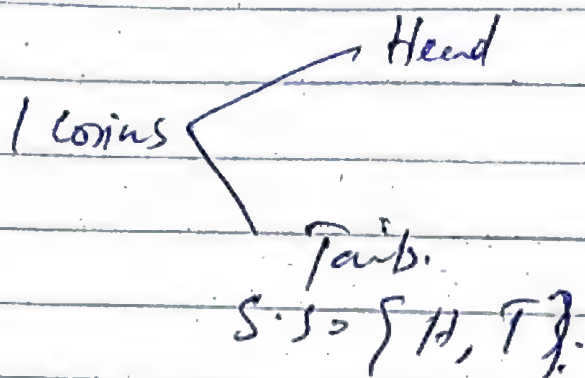
↓  
never

↓  
Always  
happen

Sample. ~~space~~ Space

A Set of all possible outcomes that can be occur

Q.1



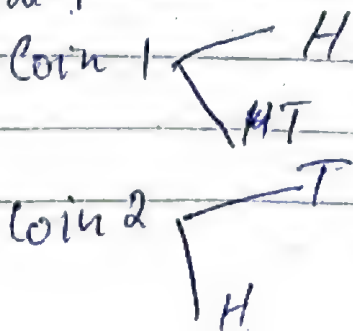
Q.2

what if we flip a two coin what are the possible outcomes what will be sample space

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$S = \{H, T, TH, TT, HH\}$$

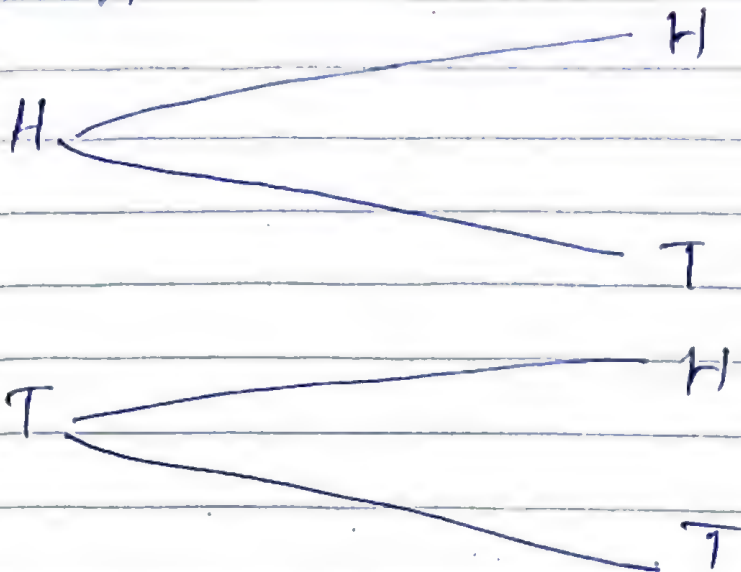
Tree diagram





coin #1

coin #2



$$= 2^n =$$

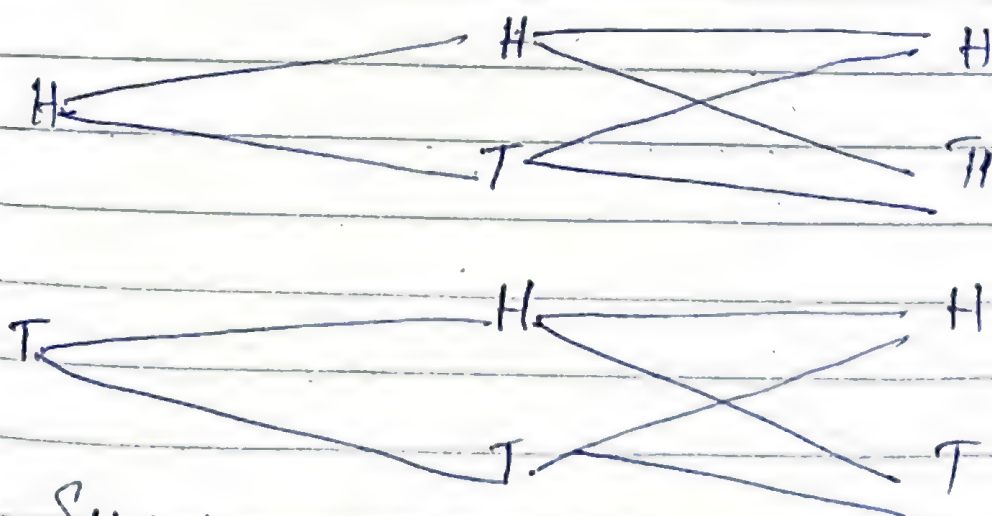
probability

3 coins

coin (1)

coin (2)

coins (3)



{H H H, H H T, H T H, H T T, T H H, T H T, T T H, T T T}

$$= 2^3 = 2 \times 2 \times 2 = 8$$

$\left\{ \begin{array}{l} H, H, H, H, H, T, H, T, H, H, T, T \\ T, H, H, T, H, T, H, T, T, T \end{array} \right\}$

### Probability: Questions

2d two fair coin a flip what  
 is probability of getting at least

(a) getting one Head

2d three coin flip what  
 probability  
 affect two tail

(b) three coins are flip. what is  
 what is the probability exactly one  
 Tail

also continued 2d tree diagram  
 continued S.S. etc



②

$$\frac{7}{8} z$$

⑤

$$\frac{3}{8}$$

③

$$7/8.$$



LC #

26/9/2024

# Probability

Q No.

A six sided die is tossed

(a) what is the probability of getting a "2"

(b) what is the probability of 3 or 5

(c) a number that is

(d) almost 4

(e) a number that is almost greater than 3.

(f) what is probability of getting number that is less than or equal to 5.

Solution

$$(a) P(2) = \frac{1}{6} = 0.166$$

(b) Probability of 3

$$P(3 \text{ or } 5) = \frac{2}{6} = \frac{1}{3}$$

Probability of 5

$$P(5) = \frac{1}{6}$$



$$S.S = \{1, 2, 3, 4, 5, 6\}$$

③ Probability of almost 4.  
 $\{1, 2, 3, 4\}$ .

$$P(X \leq 4) = \frac{4}{6} = \frac{2}{3} = 0.666$$

④ Probability of almost greater than 3.  
 $S.S \{4, 5, 6\}$ .

$$P(X > 3) = \frac{3}{6} = \frac{1}{2} = 0.5$$

⑤ less than equal 5  $S.S \{1, 2, 3, 4, 5\}$   
 $P(X \leq 5) = \frac{5}{6} = 0.83$

Q.No

A Jar contains 7 red marbles,  
6 green marbles, 5 blue marble  
and 2 yellow marbles.

(1) what is the probability of selecting  
green marbles.

(2)  $\frac{1}{10}$   $\frac{1}{10}$   $\frac{1}{10}$   $\frac{1}{10}$   $\frac{1}{10}$   
blue marbles.

(3) Green or yellow

(4) Red then blue with replacement

(5) Red then blue without replacement

(6) Red & blue without replacement

(7) what is the probability of selecting  
Red then green marble or getting  
a blue marble and then yellow  
marble with replacement



Then multiply  
are = Sum

with replacement  
Red Ball is placed again  
placed in same basket  
without replacement  
Ball pick & place in other  
basket & then take now.

a)

$$P(\text{red}) = \frac{7}{20} = 0.35$$

b)

$$P(\text{blue}) = \frac{5}{20} = 0.25$$

c)

d)

$$\frac{12.5}{20} =$$

a)

$$S.S = \{7\text{Red}, 6\text{B}, 5\text{B}, 2\text{Y}\}$$

Total marble = 20

$$= \frac{7}{20} = 0.35$$

$$c) = \frac{8}{20} = 0.4$$

$$d = 5 \times 7 = 35$$

$$\text{red } \frac{7}{20} = 0.35$$

$$\text{blue } \frac{5}{20} = 0.25$$

$$e. \frac{7}{20} = 0.35$$

$$f) \frac{5}{19} = 0.263$$

$$g) \frac{12}{20} = 0.6$$

$$\frac{12}{20} = 0.6$$

$$\frac{5}{20}$$

$$\frac{2}{20}$$

$$\frac{7}{20}$$

$$\frac{6}{20}$$

OR =

Solve by Weacher.

Color	marble Qty.
Red (R)	7
Green (G)	6
Blue (B)	5
Yellow (Y)	2
Total	<u>20</u>

$$(a) \quad P(G) = \frac{6}{20} = \frac{3}{10} = 0.30 = 30\%$$

Not selecting the green marble

$$= 1 - 0.3 = 0.7 = 70\%$$

(b)

$$P(B) = \frac{5}{20} = \frac{1}{4} = 0.25 = 25\%$$

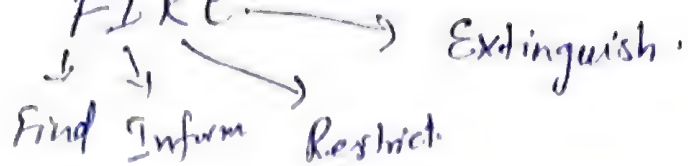
(c)

OR = Adding both probability

$$P(G) + P(Y)$$

$$\frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5} = 0.4 = 40\%$$

there is a 40% chance of selecting a B or Y marbles.



(d) Red then blue with replacement  
 it means once you take out red  
 marble on 1st try you put it back  
 for 2nd try.

$$P(R) \cdot P(B)$$

$$\frac{7}{20} \cdot \frac{5}{20} = 0.0875 = 8.75\%$$

(e) Red then blue without replacement it means  
 once you take out red marble on 1st try you  
 put separat (other basket) and then try.

$$P(R) \cdot P(B)$$

$$= \frac{7}{20} \cdot \frac{5}{19} = 0.0921 = 9.21\%$$

(f) And mean both no. will be ~~not~~ multiply  
 $P(R) \cdot P(B)$

$$= \frac{7}{20} \cdot \frac{5}{19} = 0.6 = 60\%$$

$$= 0.35 \cdot 0.263 = \overset{0.0921}{\cancel{0.6105}} = \cancel{0.6105} = 9.20\%$$



(f)

↑ order not specified

Red and Blue without replacement

$$[R][B] \text{ OR } [B][R]$$

$$\left[\frac{7}{20}\right]\left[\frac{4}{19}\right] + \left[\frac{5}{20}\right]\left[\frac{7}{19}\right]$$

$$(0.35)(0.2632) + (0.25)(0.3684)$$

$$0.0921 + 0.0921$$

$$= 0.1842 = 18.42\%$$

⑥

$$[R][G] \text{ OR } [B][Y]$$

$$\left[\frac{7}{20}\right]\left[\frac{6}{20}\right] + \left[\frac{5}{20}\right]\left[\frac{2}{20}\right]$$

$$(0.35)(0.30) + (0.25)(0.1)$$

$$0.105 + 0.025 = 0.13 = 13\%$$

$$= 13\%$$

Mutually exclusive: never come exclusively  
Simultaneously.

Q No:

Note:

\* Dice and coin throwing, are always independent event

\* They don't have any past memory.

\* they are unbiased and are all equally probable & can occur randomly.

Adding Rule:

$$P(A \text{ or } B) = P(A \cup B) \quad \text{occurring together}$$
$$= P(A) + P(B) - P(A \cap B) \quad \{P(A \cap B) \}$$

\* two events which are mutually exclusive.

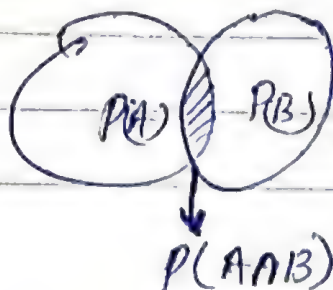
(the cannot occur simultaneously)

$$P(A \cap B) = 0$$

\* If  $A \neq B$  are different Events.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

✓  
Venn diagram.



## Multiplication Rule

- Conditional Probability (event are not independent).

$$P(A/B) = \frac{P(A \& B)}{P(B)}$$

↓

Probability of A given (B)

Multiply Both side by  $P(B)$

$$P(A/B) \times P(B) = \frac{P(A \& B)}{P(B)} \times P(B)$$

$$P(A \& B) = P(B) \times P(A/B)$$

$$P(B/A) = \frac{P(A \& B)}{P(A)}$$

Multiply by both side by  $P(A)$

$$P(A \& B) = P(A) \times P(B/A)$$



## Independent Event (IE).

An IE is that if one event does not depend on another event.

For IE

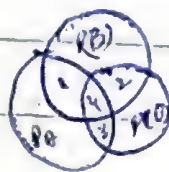
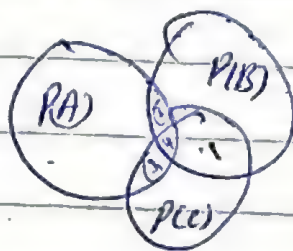
$$P(A|B) = P(A).$$

$$P(B|A) = P(B).$$

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## Additional Rules.

Apply add rule to 3 or more probabilities



$$P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(\overset{\wedge}{A \cap B}) \\ + P(\overset{\wedge}{A \cap C}) + P(\overset{\wedge}{B \cap C}) - P(A \cap B \cap C)$$

Let

4/10/2024

Q No

what is the probability of rolling a 4 or 3 when a six sided die is tossed up.

Sol

$$P(A \text{ OR } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(4) + P(3) - P(4 \cap 3)$$

$$P = \{1, 2, 3, 4, 5, 6\}$$

$$= 7 - P(\cdot)$$

$$P(A) = \frac{1}{6} + \frac{1}{6}$$

$$= \frac{2}{6} = \frac{1}{3}$$

$$P(A \cup B) = \frac{2}{6} =$$

$$P(A \cup B) = \frac{1}{3}$$

S/v.

$$P[3 \cup 4] =$$

$$P(3) = \frac{1}{6}$$

$$P(4) = \frac{1}{6}$$

$$P(3 \cap 4) = 0$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{6} + \frac{1}{6} - 0$$

$$= \frac{2}{6}$$

$$P(A \cup B) = \frac{1}{3}$$



Q. No.

Q. No.:

A Bag consist of 8 Red ball marbles,  
7 blue marble, 6 green marbles and  
4 yellow marbles.

What is the probability of  
Selecting.

(1) A red marble

(2) A Blue marble

first try and then a green marble  
on second try with replacement

(3) the yellow marble on the first try & then  
a red marble on the second try ~~on red~~ without replac

(4) two blue marbles with replacement

(5) two green marble without replacement

Soln.

Color	Qty of marble
Red	8
Blue	7
Green	6
Yellow	4
25 marble	

(a) Probability of Red marble

$$P(R) = \frac{8}{25} = 0.32 = 32\%$$

32% chance of selecting a red marble on

(b) first try.

$$P(B) = \frac{7}{25} = 0.28$$

with replacement Green

$$P(G) = \frac{6}{25} = 0.24$$

$$P(BG) = \frac{7}{25} \times \frac{6}{25} = 0.067 = 6.72\%$$

(c)

$$P(Y) = \frac{4}{25} = 0.16$$

without replacement Red.

$$P(R) = \frac{8}{24} = 0.333$$

(c)

$$\begin{aligned} P(YR) &= \frac{4}{25} \times \frac{8}{24} \\ &= 0.16 \times 0.333 \\ &= 0.053 \\ &= 5.3\% \end{aligned}$$

(d) two blue marble with replacement  
= ;

$$P(A \cup B) = P(A) + P(B) - P(AB)$$
$$P(A \cup B) = 0.28 + 0.28 - 0.0784$$

$$\frac{7}{25} \times \frac{7}{25}$$

$$P(AB) = 0.28 \times 0.28 = 0.0784 = 7.84\%$$

② Two green marble without replacement

$$\begin{aligned} &= \frac{6}{25} \times \frac{6}{24} \\ &= 0.24 \times 0.25 = 0.06 = 6\% \\ &= 0.06 \end{aligned}$$

$$= \frac{6}{25} \times \frac{3}{24}$$

$$0.24 \times 0.125$$

$$P(GG) = 0.05 = 5\%$$



Ques Last year mid term Question

Ans:

The Reliability of a missile is 0.85. If a Salvo of two missile is fired.

What is the probability of at least one hit

Note:

Assume independence of missile hits

Solve the problem using

(a) Probability Equation for two independent event

(b) Sequential tree Diagram Construction

(c) Basic Probability technique

Hint:

Probability of A or B occurring if A & B are independent.

$$P(A+B) = P(A) + P(B) - P(A)P(B)$$

$$P(\text{missile 1}) = P(M_1) = 0.85$$

$$P(\text{missile 2}) = P(M_2) = 0.85$$

Soln (Teacher).

Let

A be the event for 1<sup>st</sup> missile hit

B be event for 2<sup>nd</sup> missile hit

So

$$P(A) = P(B) = 0.85$$

Failure.  $P(\bar{A}) = P(\bar{B}) = 0.15$

$$P(A \cup B) = P(A) + P(B) - P(A)P(B)$$

$$= (0.85) + (0.85) - P(0.85)(0.85)$$

$$P(A \cup B) = 0.9775 \text{ Ans}$$

$$P(A \cup B) = 97.75\%$$

$$\textcircled{b} S.S = \{ HH, HM, MH, MM \}$$

there are 4 possible outcomes.

$$\{ AB, \bar{A}\bar{B}, \bar{A}B, A\bar{B} \}$$

for possibility of atleast 1 hit X

=

$$= \frac{3}{4} = 0.75 = 75\%$$

Now lets take the probability  
of both missile.

$$P(A) \times P(B) = P(\bar{A} \cdot \bar{B})$$

$$[0.15] \times [0.15]$$

$$P(\bar{A} \cdot B) = 0.0225 = 2.25\%$$

Probability of atleast one hit

$$P(S) = 1 - P(+)$$

$$= 1 - 0.0225$$

$$P(S) = 0.9775$$

$$P(S) = 97.75\%$$



2. oil Probability: Quality Reports  
oil losses.

(C) Sequential Tree diagram.

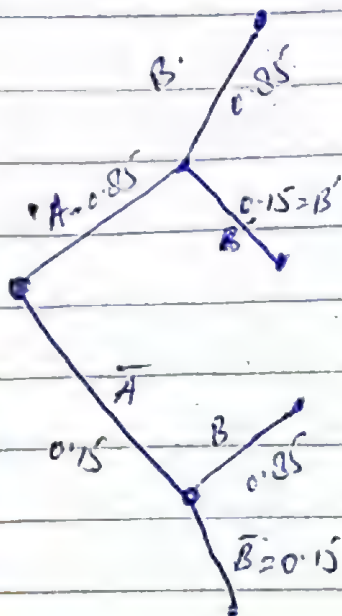
$\{AB, AB, AB, AB\}$

0.7225 0.1275 0.1275 0.0225

rough  
work

$= 0.7225 + 0.1275 + 0.1275$

$= 0.9775$



$= 1 - 0.9775$

$= 0.0225$  miss

$A, B = \text{mutually exclusive}$

$$P(AB) = 0.85 \times 0.85 = 0.7225$$

$$P(A\bar{B}) = 0.85 \times 0.15 = 0.1275$$

$$P(\bar{A}B) = 0.15 \times 0.85 = 0.1275$$

$$P(\bar{A}\bar{B}) = 0.15 \times 0.15 = 0.0225$$

Probability of hit divided by.  
Summing the product of each  
path which leads to at least  
one hit As each path are (M.E)

$$P(AB) + P(A\bar{B}) + P(\bar{A}B) \\ = 0.7225 + 0.1275 + 0.1275$$

$$P(\text{System}) = 0.9775$$

Note:

$$P_1 + P_2 + P_3 + P_4 + \dots + P_n = 1$$

For

QNO

Sarah is deciding which course she wants to take in her next college semester. Probability that she enrolls in an Algebra course is 0.3 and probability that she enrolls in a biology course is 0.7. Probability that she will enroll in ~~Algebra~~ <sup>Biology</sup> course given that she will enroll in biology course is 0.4

- (a) What is the probability that she will enroll in both an Algebra & Biology course
- (b) What is the probability that she will enroll in an algebra course or a biology course
- (c) Are the two events independent (proof that).
- (d) Are the two events mutually exclusive.



$$P(A \& B) = P(B/A) \times P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(a)

$$P(A) = \text{Algebra} = 0.3$$

$$P(B) = \text{biology} = 0.7$$

$$P(A \cap B) = 0.4$$

$$P(A) = \frac{0.3}{1} = 0.3 \quad \text{Rough work}$$

$$P(A \& B) = (0.4)(0.3) = 0.12$$

Sol. (Teacher)

$$P(A) = 0.3$$

$$P(B) = 0.7$$

$$P(A/B) = 0.4$$

(a)

$$P(A \& B) = P(A/B) \times P_B$$

$$= 0.4 \times 0.7$$

$$P(A \& B) = 0.28 = 28\%$$

$$\textcircled{b} \quad P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$$= 0.3 + 0.4 - 0.28$$

$$= 0.42$$

$$P(A \cup B) = 0.42 = 42\%$$

$\textcircled{c}$

Are these two events independent

$$P(A|B) = P(A).$$

$$0.4 > 0.3$$

$$0.4 \neq 0.3$$

↓ independent

Events are not independent

$\textcircled{d}$

Are the two events mutually exclusive we use

$$P(A \text{ and } B) = 0$$

But we know that

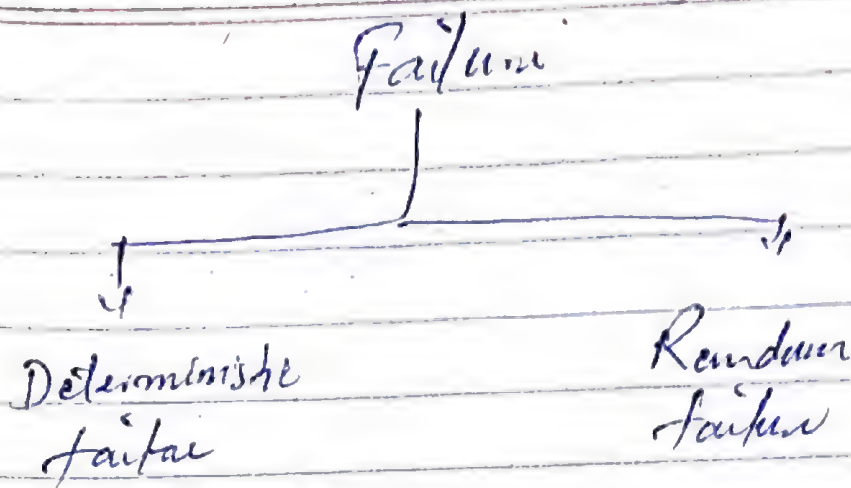
$$P(A \text{ and } B) \neq 0$$

$$P(A \cap B) = 0.28$$

Event  $(A \cap B)$  are not mutually.

Exclusive event.





Random failure:

They may exhibit a pattern that can be modelled by some probability distribution

Random variable:

They allow us to pass from experimental outcome to a numerical function of the outcome.

Types of random variable  
(location unknown).

- ① discrete random variable
- ② continuous random variable

(Temp, mass,  
etc)

Standard deviation
Sample position $n/x$

L # C.

FMEA

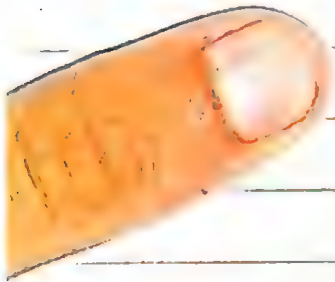
Failure mode & effect analysis.

↓  
Design  
Failure  
Mode & Effect  
analysis.

(DFMEA)

↓  
Process  
Failure  
Mode & Effect  
analysis.

(PFMEA)



mid term Paper / Rattent  
// MTFA vs MTBF  
Q.

-  $R(t) = 1 - \text{failure}$

~~What the~~ Difference b/w following

- Quality vs Reliability

MTTF vs MTBF

- Name potential ways to improve reliability.

- why product fail.

Q

Don't one put on test

MTTF:

calculate the failure rate

Q both sub, come. Sketch  
Explain the three main failure  
rate feature

$$f(t) = \begin{cases} 0.0001e^{-0.0001t} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Q R(t) =

Q MTTF

Q median



# Paper Pattern

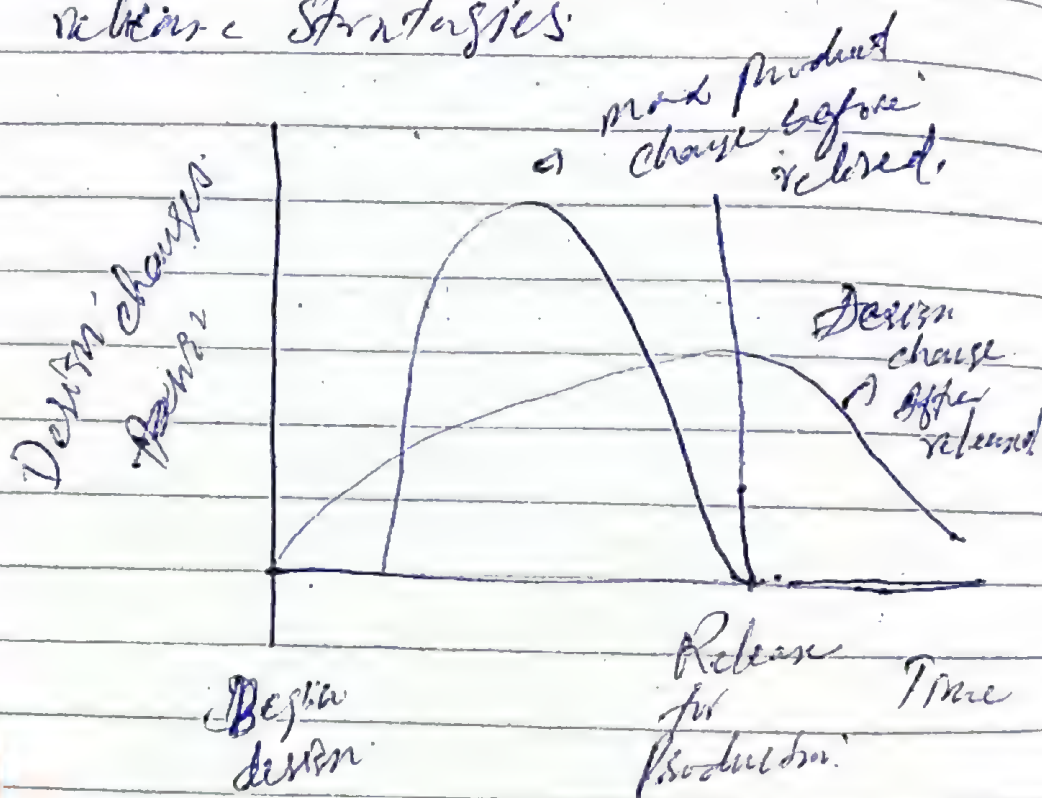
Proof mathematical MTP.

$$MTP_2 = \int_0^{\infty} R(t) dt$$

~~Rate~~

Details

D. Release Strategies



# Mid life upgradation (MLU)

## • FMEA :-

- Technique Risk Assessment prior to release of design, process or service

Severity  
Occurrence

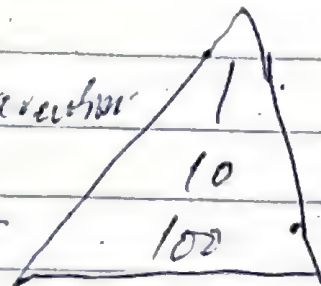
Probability of detection in the event of occurrence

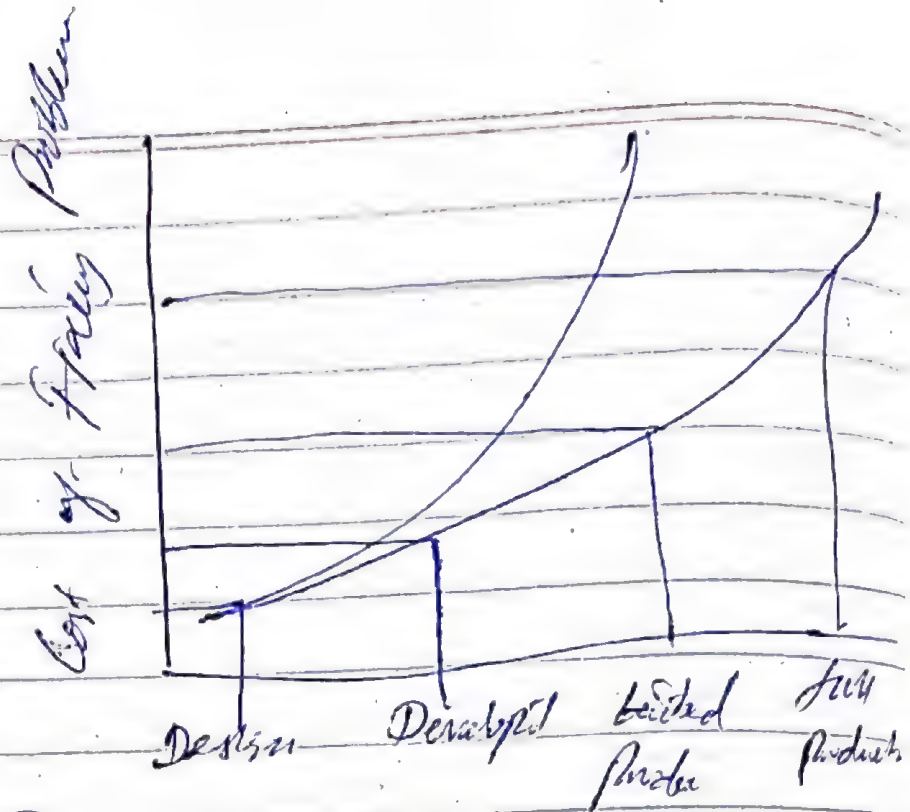
RPN  $\rightarrow$  Number  $\rightarrow S \times O \times D$   
 $\downarrow$   
Risk Priority  $10 \times 10 \times 10$

The

1 - 10 - 100 Rule

one dollar spent on prevention  
10 dollar on correction  
100 dollar on failure cost





Stage ①

① cost of design fixation is less.

Cost of Development is more cost as compared to design. Stage ②

Stage ③

Entered production cost is high. at this stage and further at this stage the fixation cost of very high bc they design the production machines for this product.

Stage ④

full production the fixate at machine design.

Dies, fixture has produced machines are design already assembly has design.



CQI = Continuous Quality Improvement

High viscosity
Ball rotation

Benefits of PMDA.

- ① Higher reliability.
- ② Better quality
- ③ increased safety,
- ④ Enhanced customer ~~off~~ satisfaction
- ⑤ contributes to cost saving
- ⑥ Decrease development

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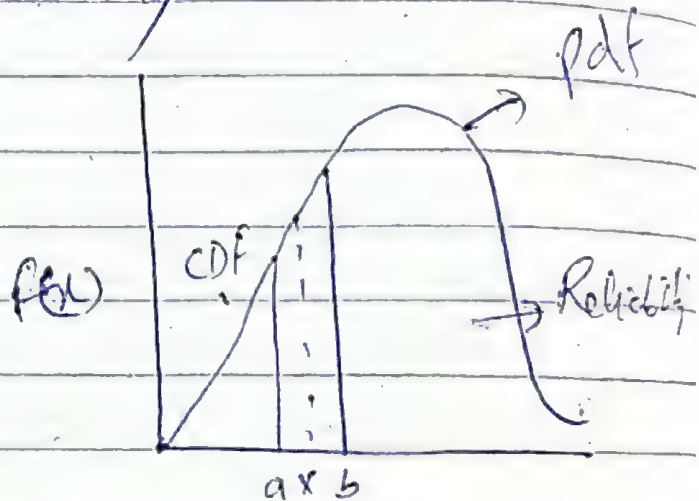
DPMEA Roadmap. (Note)  
Quality Function Development:

L.C.#  
- Random variable.

Redundancy  $\rightarrow$  Backup  $\rightarrow$  Standby.

- Discrete  
- Continuous

① Probability Density Function (PDF)

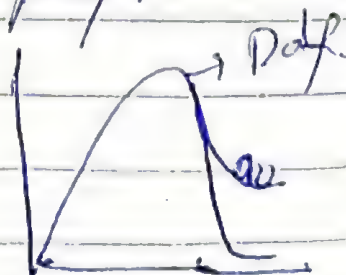


② Cumulative Function.

③ Reliability function.  $R(t)$

Probability Density Function (PDF).

for  $f(x)$  to be a pdf.



Exponential Reliability

$$\left. \begin{aligned} \lim_{x \rightarrow \infty} f(x) &= 0 \\ \lim_{x \rightarrow \infty} f'(x) &= 0 \\ \lim_{x \rightarrow \infty} \frac{1}{x} &= 0 \end{aligned} \right\}$$

Example (pdf)

$$f(x) = \begin{cases} c(4x - 2x^2) & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

$$f(x) = \int_0^2 4x - 2x^2$$

~~$f(x) = 4 - 4x$~~   
~~Apply the limits.~~  
 ~~$f(x) = 0$~~   
 ~~$f(x)$~~

$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^0 f(x) dx + \int_0^2 f(x) dx + \int_2^{\infty} f(x) dx = 1$$

Sol

$$= \int_0^2 c(4x - 2x^2) dx = 1$$



Example:

Time to failure (in) hr)  
for a capacitor has the following  
pdf what is the probability of  
failure by  $t = 200$  hr?

$$f(t) = 0.01 e^{-0.001t}$$

Since its time it must

$$F(x) = \int_{-\infty}^x f(x) dx$$

$$= \int_0^{200} 0.01 e^{-0.001t} f(t) dt$$

$$P(t \leq 200) = \int_0^{200} 0.01 e^{-0.001t} dt$$

Formulas:

$$F(x) = \int_{-\infty}^x f(y) dy$$

$$R(x) = \int_x^{\infty} f(y) dy$$

$$f(x) = \frac{dF(x)}{dx}$$

$$F(x) = 1 - R(x).$$

$$\frac{dF(x)}{dx} = \frac{d(1 - R(x))}{dx} = -\frac{dR(x)}{dx} = f(x).$$

# Why Improve Reliability

## potential ways to Improve Reliability

Failure rate or Hazard,

$$\lambda = \frac{f(t)}{R(t)}$$

Reliability function

~~Example~~

Prove 2

$$1 - R(t) = F(t)$$

$$\lambda(t) = \frac{f(t)}{R(t)}$$

$$\frac{\frac{dF(t)}{dt}}{R(t)} = \frac{\frac{d[1-R(t)]}{dt}}{R(t)} = \frac{-\frac{dR(t)}{dt}}{R(t)}$$

$$\lambda(t) dt = \frac{dR(t)}{R(t)}$$

$$\int_0^t \lambda(t) dt = \int_1^{R(t)} \frac{dR(t)}{R(t)} = -\ln R(t)$$

$$R(t) = e^{-\int_0^t \lambda(t) dt}$$



Expt  
1.2.4.4.

$R(t) = \lambda(t)$  Reliability  
meth.

(22/11/24)

Problem:-

Given the Hazard function when  $t$  is measured in operating hours.

What is the design life if a 0.98 reliability is desired  
Take Hazard function.

$$\lambda(t) = 5 \times 10^{-6} (t)$$

Sol:-

$$-\int_0^t \lambda(t) dt$$

$$R(t) = e$$

$$-\int_0^t 5 \times 10^{-6} t dt$$

$$= e$$

$$R(t) = e^{-\left[\frac{5 \times 10^{-6} \times t^2}{2}\right]_0^t}$$

$$0.98 = e^{-[2.5 \times 10^{-6} t^2]}$$

$$\ln 0.98 = \ln e^{-[2.5 \times 10^{-6} t^2]}$$

$$\ln 0.98 = 2.5 \times 10^{-6} t^2 \times 1$$

$$t^2 = \frac{0.0202}{2.5 \times 10^{-6}}$$

$$t = 89.88 \text{ hrs}$$

$$f(x) = \text{pdf}$$

$$F(x) = \text{CDF}$$

$$f(x) = \text{pdf}$$

Problem:

pdf

$$\text{Given that } f(x) = 0.048x[5-x]$$

- (a) Verify that  $f$  is a pdf
- (b) What is the probability that " $x$ " is greater than 4
- (c) What is the probability that  $x$  is in b/w 1 & 3 inclusive inclusive.

Sol.

$$\int_{-\infty}^{+\infty} f(x) dx = 1 \rightarrow \boxed{\text{formula}}$$

↓

integral from  $-\infty$  to  $+\infty$  for  
any pdf = 1

Sol.

$$\int_{-\infty}^{+\infty} f(x) dx = 1$$

(a)

$$f(x) = 0.048x(5-x)$$

$$\text{for } \int_{-a}^{+a} 0.048x(5-x)$$

(a)

if

$$f(x) = 0$$

$$0.048(0)[5-0]$$

$$f(x) = 0$$

(a)

if

$$f(x) = 5$$

$$= 0.048(5)[5-5]$$

$$f(x) = 0$$

So  $x'$  must be 4w  $0 \leq x \leq 5$

It will give the result  
So if we integrate from

$$\int_0^5 f(x) dx = 1$$



$$= \int_0^5 0.045(x)(5-x) dx$$

$\Rightarrow$  So  $f(x)$  is a Pdf.

$$(b) P(4 \leq x < 5) = \int_4^5 0.048x(5-x) dx$$

$$P(4 \leq x < 5) = 0.104 = 10.4\%$$

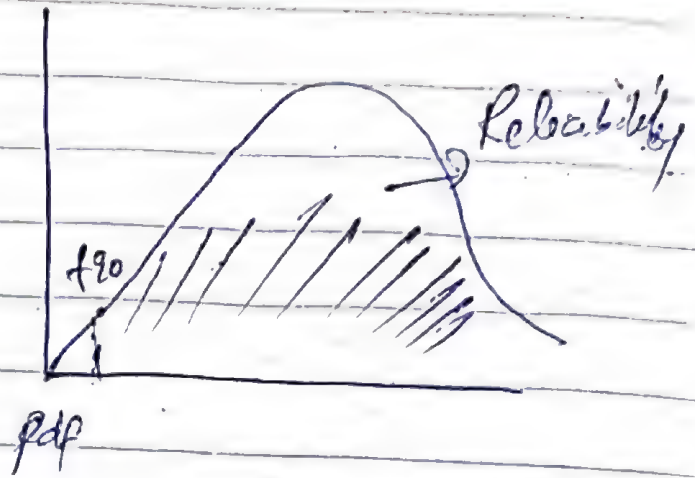
$$(c) P(1 \leq x < 3) = \int_1^3 0.048x(5-x) dx.$$

$$54.4\%$$

prob.

## Design Life

a



~~Very Imp~~ design life  $t_d$  calculate

Reliability

Problem:

Given the following Pdf. for the random variable  $T$  the time in operating hr) to failure of a compressor. what is its reliability for a 100 hr. operating life? what is the design life for 95% reliability.

Solution

$$f(t) = \begin{cases} 0.001 \\ (0.001t + 1)^{-2} \end{cases} \quad t \geq 0$$

$$f(t) = \frac{0.001}{(0.001t + 1)^2} \quad t \geq 0$$

$$R(t) = \int_t^{\infty} \frac{0.001}{(0.001t + 1)^2} dt$$

Let  $u = 0.001t + 1$

$$\frac{du}{dt} = 0.001$$



$$\frac{du}{0.001} = dt$$

$$= \frac{0.001}{0.001} \int_t^{\infty} \frac{du}{u^2}$$

$$= \int_t^{\infty} \frac{du}{(u)^2}$$

$$= \int_t^{\infty} u^{-2} du$$

$$= \left[ \frac{u^{-2+1}}{-2+1} \right]_t^{\infty}$$

$$= \left[ \frac{1}{u} \right]_t^{\infty}$$

$$\left[ -\frac{1}{\infty} + \frac{1}{u} \right]$$

$$0 + \frac{1}{u}$$

$$0.7 \frac{1}{U}$$

$$R(t) = \frac{1}{(0.001t + 1)}$$

$$0.95 = \frac{1}{(0.001t + 1)}$$

$$(0.001t + 1) \cdot 0.95 = 1$$

$$R(100) = \frac{1}{[0.001 \cdot 100 + 1]}$$

$$R(100) = 0.909$$

(b) Design life for 95% reliability

$$0.95 = \frac{1}{0.001t + 1}$$

$$0.95 \times (0.001t + 1) = 1$$



$$0.00095t + 0.95 = 1$$

$$0.00095t = 1 - 0.95$$

$$t = \frac{0.05}{0.00095}$$

$$t \approx 52.631$$

$$t \approx 52.631 \text{ hrs.}$$

Median

Exact median of  
an function = 50th percentile.

Problem:

consider the following pdf with  
 $X$  in hrs.

$$f(x) = \begin{cases} \frac{3}{8} (4x - 2x^2), & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

other with median find median

$$F(x) = R(x) = 0.5$$

$$F(x) = \int_0^x \frac{3}{8} (4t - 2t^2) dt = 0.5$$



$$F(x) = -\frac{3}{8} \left( 2x^2 - \frac{2}{3}x^3 \right) = 0.5$$

$$F(t_{50}) = \frac{1}{2}$$

$$f(x) = R(x) = 0.5$$

$$= \int_0^x \frac{3}{8} [4t - 2t^2] dt = \frac{1}{2}$$

$$\frac{3}{8} \left[ 2t^2 - \frac{2}{3}t^3 \right]_0^x = \frac{1}{2}$$

$$2x^2 - \frac{2}{3}x^3 = \frac{8}{3}$$

$$-\frac{2}{3}x^3 + 2x^2 = \frac{4}{3}$$

$$= 2.736 \quad \times \quad 0 < x < 2$$

$$= 0.99505$$

$$= -0.731 \quad \times$$

0 ..

